A Simple 3-Step Audit Sample Size Calculation for Election Contests

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The following summary describes how a Secretary of State's Office, State Board of Elections or independent audit team could calculate the minimum number of precincts that need to be randomly selected for each contest in order to achieve a specified statistical power of detecting vote miscount if miscounts are large enough to alter the outcome. It may be used for federal and statewide contests (e.g., President, US Senator, US Representative, Governor, Secretary of State, statewide ballot propositions) as well as contests in individual state legislative districts (e.g., State Senate, State Assembly), towns or counties. When combined with targeted non-random audits of precincts in which candidates and/or the audit team believe there may be miscounted votes, it can serve as a framework for a statistically accurate, fair and efficient audit protocol.

Introduction

As described in Appendix B of [McCarthy et al, 2007], calculation of the sample size (i.e., n = number of precincts) necessary to detect a miscount large enough to alter the outcome of a particular contest requires the following parameters:

P: the desired probability of sampling at least one miscounted precinct, if there are enough miscounted precincts to alter the outcome -- a number less than 1 but usually at least .9;

***N*:** the total number of precincts in the contest (i.e., where the race appears on the ballot);

Bmin: the smallest number of precincts that would have to be miscounted in order to change the outcome, taking into account the amount of variation between precincts with larger and smaller numbers of votes (i.e., fewer large precincts with miscounts would be required to reverse the outcome); and

WPM: the Within-Precinct Miscount, the assumed *maximum miscount per precinct* that the random sample must detect. For the purpose of calculating sample sizes, we assume that no more than 20% of the total vote could be switched from one candidate or ballot position to another in each precinct without arousing suspicion and thus triggering a targeted, non-random selection of such precincts to be audited. (Switching 20% of the total vote represents a 40-point shift in the victory margin percentage within that precinct.) We refer to this value as the Within-Precinct Miscount (WPM). Thus, if all precincts contained the same number of votes, a 10-percentage-point margin could be overcome by switching 20% of votes in 25% of the precincts. More generally, any margin can be overcome by switching 20% of the votes in precincts containing enough votes to equal 2.5 times the margin.[1]

Note that *Bmin* depends on WPM. Strictly speaking, miscounts *larger* than WPM in *fewer* than *Bmin* precincts could also alter the electoral outcome; hence it is also important to provide for targeted non-random precinct audits to confirm that such miscounts have occurred.

[1] For example, suppose that a contest spans 400 precincts with 500 votes each, and the margin is 1,000 votes. Then to change the outcome would require miscount in five precincts containing a total of 2,500 votes, with 20% of these votes having been shifted from the apparent loser to the apparent winner. However, if some precincts contain more votes than others (as in real life), fewer than five large precincts may contain a total of 2,500 votes.

3-Step Procedure

1. The minimum number of precincts to change the outcome of any election as reported by the voting system (*Bmin*) shall be calculated as follows:

For any contest with a single winner, the margin of victory between the two candidates receiving the largest number of votes as reported by the voting system shall be calculated.[2] All precincts used in each contest shall be sorted in descending order by the total number of votes cast in the contest (including overvotes, undervotes, or their equivalents) in each precinct. Beginning with the precinct with the largest vote count, the minimum number of precincts containing at least 2.5 times the margin of votes shall be determined. This result is *Bmin*.

2. The number of precincts to be audited in each contest (*n*) shall be equal to:

$$(N - (Bmin - 1) / 2) \times (1 - (1 - .99)^{1/Bmin})$$

rounded up to the next greater integer, where *N* is the total number of precincts, *Bmin* is the minimum number of miscounted precincts to change the outcome, and .99 is the statistical power (*P*). (This power value can be changed if necessary.) This formula is drawn from [Aslam, Popa and Rivest 2007]. (Note: the preceding equation to calculate the sample size can be solved or checked on a hand calculator and is therefore *not* dependent on software.)

3. Each county or equivalent jurisdiction shall audit its pro rata share of the total number of precincts to be audited for each contest (*n*) as determined by multiplying *n* by:

$$\frac{\text{the total number of precincts in the contest which reside in the county}}{N}$$

and rounding the product up to the next greater integer. (*N* is the total number of precincts in which the contest appeared on the ballot in *all* counties.) Due to rounding, the sum of these pro rata shares may be greater than the sample size calculated in step 2.

Example

For example, if *N* = 500, *Bmin* = 50 and we want a statistical power of 99% (.99), we could write and solve the equation in Step 2 as:

$$n = (500 - (50 - 1) / 2) \times (1 - (1 - .99)^{1/50})$$
$$= (500 - 24.5) \times (1 - (.01)^{.02})$$
$$= 475.5 \times .0878$$
$$= 42 \text{ precincts (when rounded up to next whole precinct)}$$

If one county contained 200 of these 500 precincts, its pro rata share would then be 42 x (200 / 500) = 16.8, rounded up to 17 precincts.

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[2] For contests with more than one winner, the margin is the vote difference between the winner receiving the fewest votes and the first runner-up.

Note that as the number of precincts required to potentially change the outcome of a contest (*Bmin*) becomes smaller, the number of sample precincts required (*n*) increases.

Stanislevic has developed an Excel spreadsheet to demonstrate and automate Steps 1 and 2 of the above process using precinct-level vote count data for up to 5,000 precincts (expandable by Copy and Paste). Per Step 1, the user must sort the precincts by vote count (Excel can be used to do this) and the worksheet generates the sample size and random precinct selections (assuming a single jurisdiction), including a dynamic chart. The results can be confirmed manually using the above equation, and the random precinct selections can be performed without a computer for greater transparency. The spreadsheet can be downloaded here: https://vvf.jot.com/WikiHome/PublicDocuments/GraphicPaper/PrecinctDistUpTo5000.xls

Estimation of *Bmin*

The preferred method of calculating the value of *Bmin* (i.e., taking precinct size variations into account) is to do so directly if precinct-level vote counts are available from a preliminary report of precinct-level election returns, but this may not always be possible. In that event, *Bmin* can be estimated based on the distribution of precinct-level votes in one or more previous elections, or by using a heuristic approximation based on other typical precinct-level data. A heuristic that can approximate the distribution of precinct-level vote counts is the equation:

$$B_{min} = \lceil \frac{B}{log_{10}(N/B) + 1} \rceil$$

where *Bmin* and *N* are as above, and *B* is the minimum number of miscounted precincts that would change the outcome if all precincts had the same vote count (or if all miscounts were in average-sized precincts). The half-brackets above represent the ceiling (round-up) function. For more information, see Appendix B of [McCarthy, et al, 2007].

Additional Work

Slightly larger sample sizes may be required to rule out outcome-altering miscounts in the presence of routine errors. We will discuss this separately. Note that no election audit protocol is complete without rules about what to do when one or more miscounts are detected in the initial sample.

References

[McCarthy, et al, 2007] McCarthy, John, Howard Stanislevic, Mark Lindeman, Arlene Ash, Vittorio Addona and Mary Batcher, Verified Voting Foundation, "Percentage-based versus SAFE Vote Tabulation Auditing: A Graphic Comparison", July 26, 2007 (Revised Oct. 2, 2007), Published on the Internet at: http://www.verifiedvotingfoundation.org/auditcomparison

[Aslam, Popa and Rivest 2007] Aslam, Javed A., Raluca A. Popa and Ronald L. Rivest. 2007. "On Estimating the Size and Confidence of a Statistical Audit." April 22, 2007. Published on the Internet at: http://theory.lcs.mit.edu/~rivest/AslamPopaRivest-OnEstimatingTheSizeAndConfidenceOfAStatisticalAudit.pdf