

How to Calculate Sample Sizes (to detect outcome-altering miscounts)

- Three-Step Procedure for a Statistically Accurate, Fair and Efficient (SAFE) Audit

For each contest on the ballot to be audited:

- 1. Determine the *smallest number* of precincts that could change the outcome (by switching $\frac{1}{2}$ the margin).
- 2. Calculate the sample size.
- 3. Each county audits its pro rata share of sample precincts.

3-Step Procedure

- 1. Determine the *smallest number of precincts that could change the outcome.*
 - Sort precincts by vote count in descending order. Include all ballots cast for each race, including undervotes.
 - Use WPM (or another method) to determine maximum vote switch in each precinct, e.g. 20%.
 - Find minimum number of precincts that can contain the switched votes (largest precincts).

3-Step Procedure (continued)

- 2. Calculate the sample size (n).*



$$n = (N - (Bmin - 1) / 2) \times (1 - (1 - P)^{1/Bmin})$$

- $Bmin$ = the result from Step 1.
- N = the number of precincts where the contest appeared on the ballot (all counties).
- P = the statistical power (probability) of detecting at least 1 miscounted precinct.

* Equation from, Aslam, Popa and Rivest, "On Estimating The Size And Confidence Of A Statistical Audit", <http://theory.lcs.mit.edu/~rivest/AslamPopaRivest-OnEstimatingTheSizeAndConfidenceOfAStatisticalAudit.pdf>

3-Step Procedure (continued)

- 3. Each county audits its pro rata share of sample precincts.
 - Each county or equivalent jurisdiction audits its pro rata share of the total number of precincts to be audited for each contest.
 - Multiply n by:

$$\frac{\text{total precincts in the contest in the county}}{N}$$

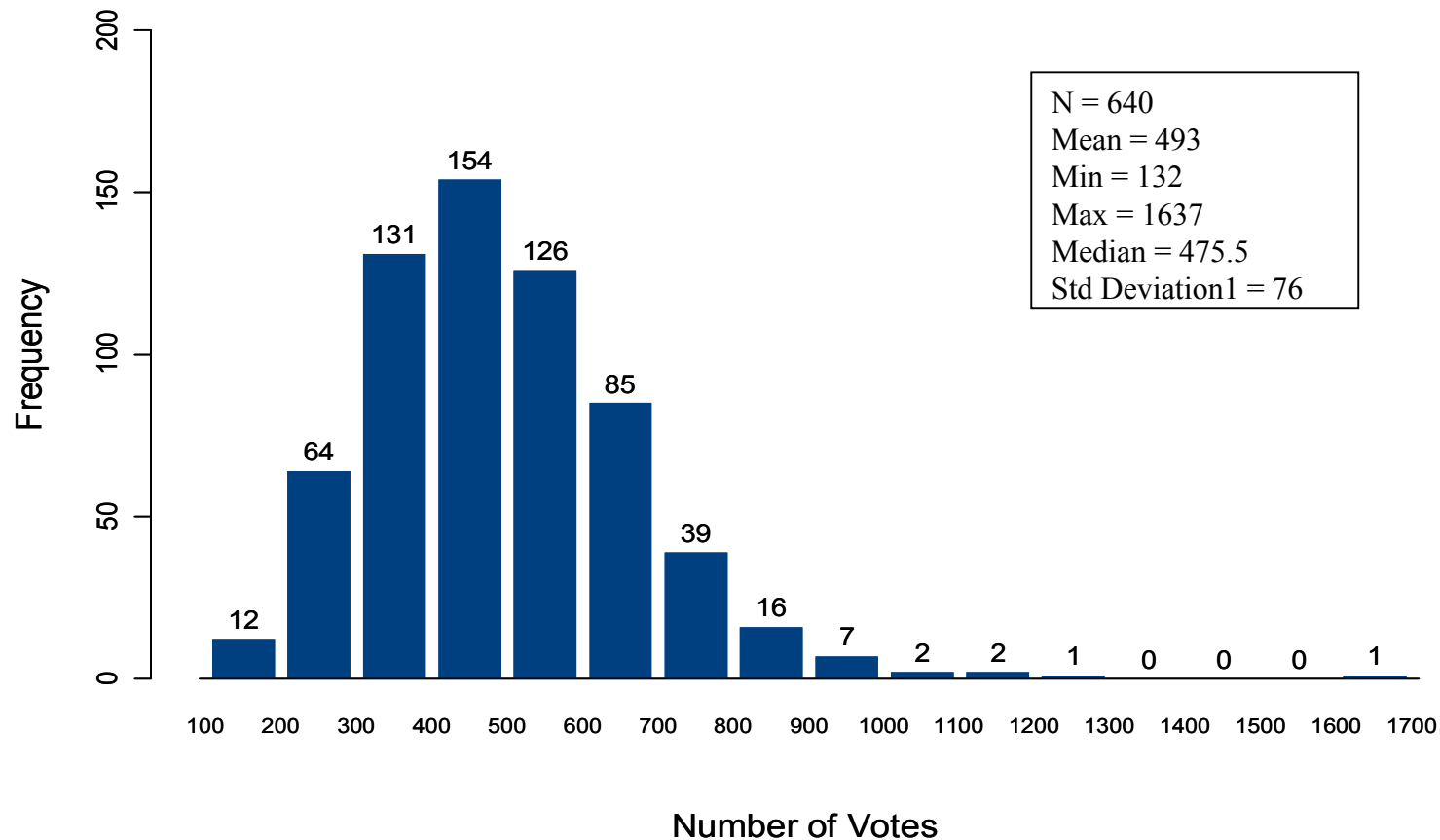


And

Investigate Discrepancies!



Distribution of Votes Counted in 2004 among the 640 Precincts of Ohio's Fifth Congressional District



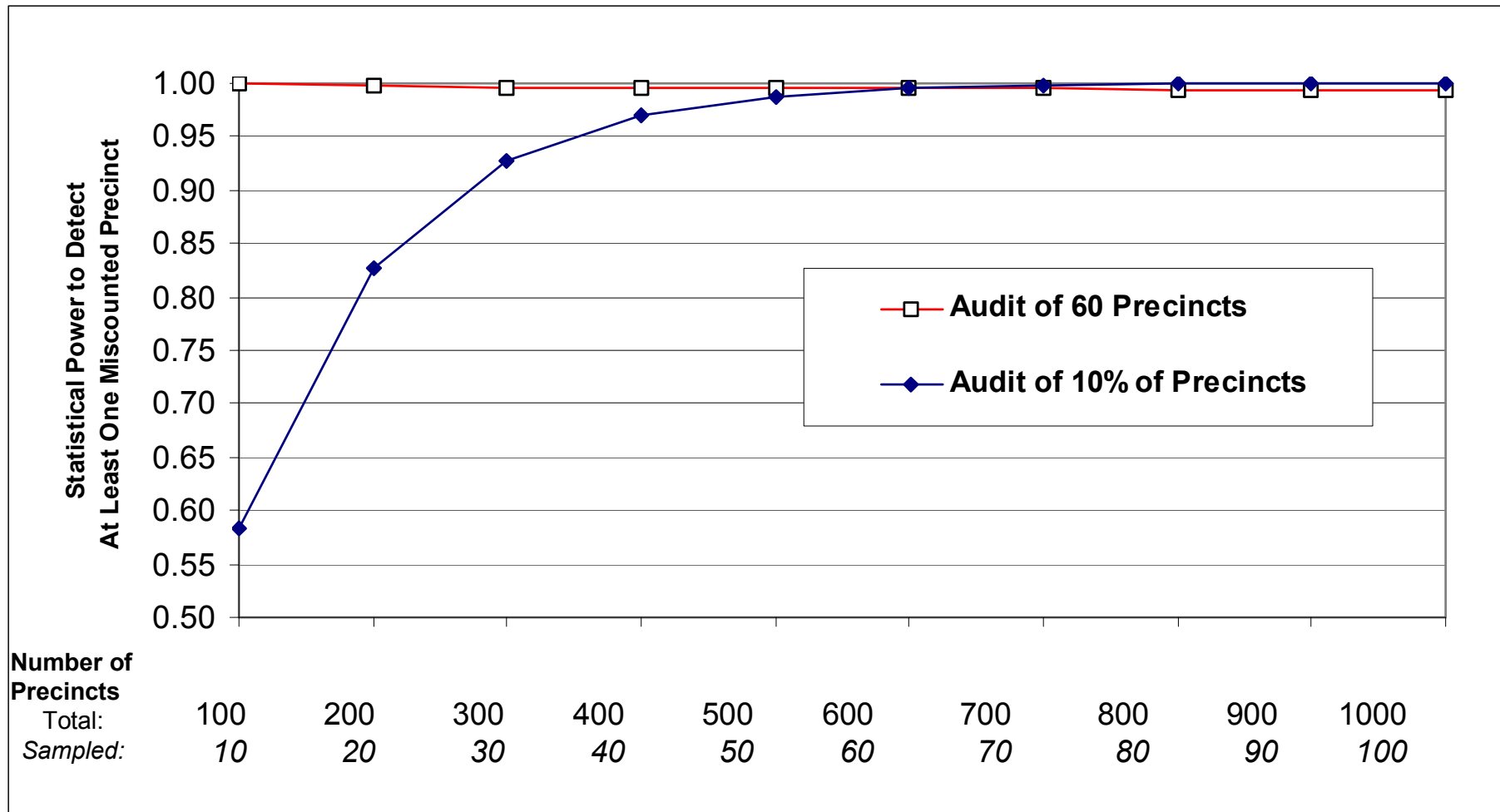
When vote miscounts are concentrated in larger precincts, fewer miscounted precincts are needed to alter an election. Consider an election with a winning margin of 6 points (for instance, 53% to 47%). This margin could be overturned by 20% shifts in precincts containing 15% of all votes

Federal Elections (2002-2006) Total Hand-Counted Votes by Type of Audit

Type of Audit	Percentage-based				SAFE	
	Tiered 3-5-10%	2%	3%	10%	99% power	95% power
<i>Power of the Audit</i>	<i>Number of elections (percent)</i>					
at least 99%	1152 (82.7%)	1089 (78.2%)	1152 (82.7%)	1292 (92.7%)	1393 (100%)	-
from 95% up to 99%	77 (5.5%)	98 (7.0%)	74 (5.3%)	31 (2.2%)	-	1393 (100%)
from 50% up to 95%	112 (8.0%)	137 (9.8%)	110 (7.9%)	51 (3.7%)	-	-
less than 50%	52 (3.7%)	69 (5.0%)	57 (4.1%)	19 (1.4%)	-	-
<i>Total hand-counted votes (in millions)</i>	20.5	15.3	19.4	57.6	23.0	19.0

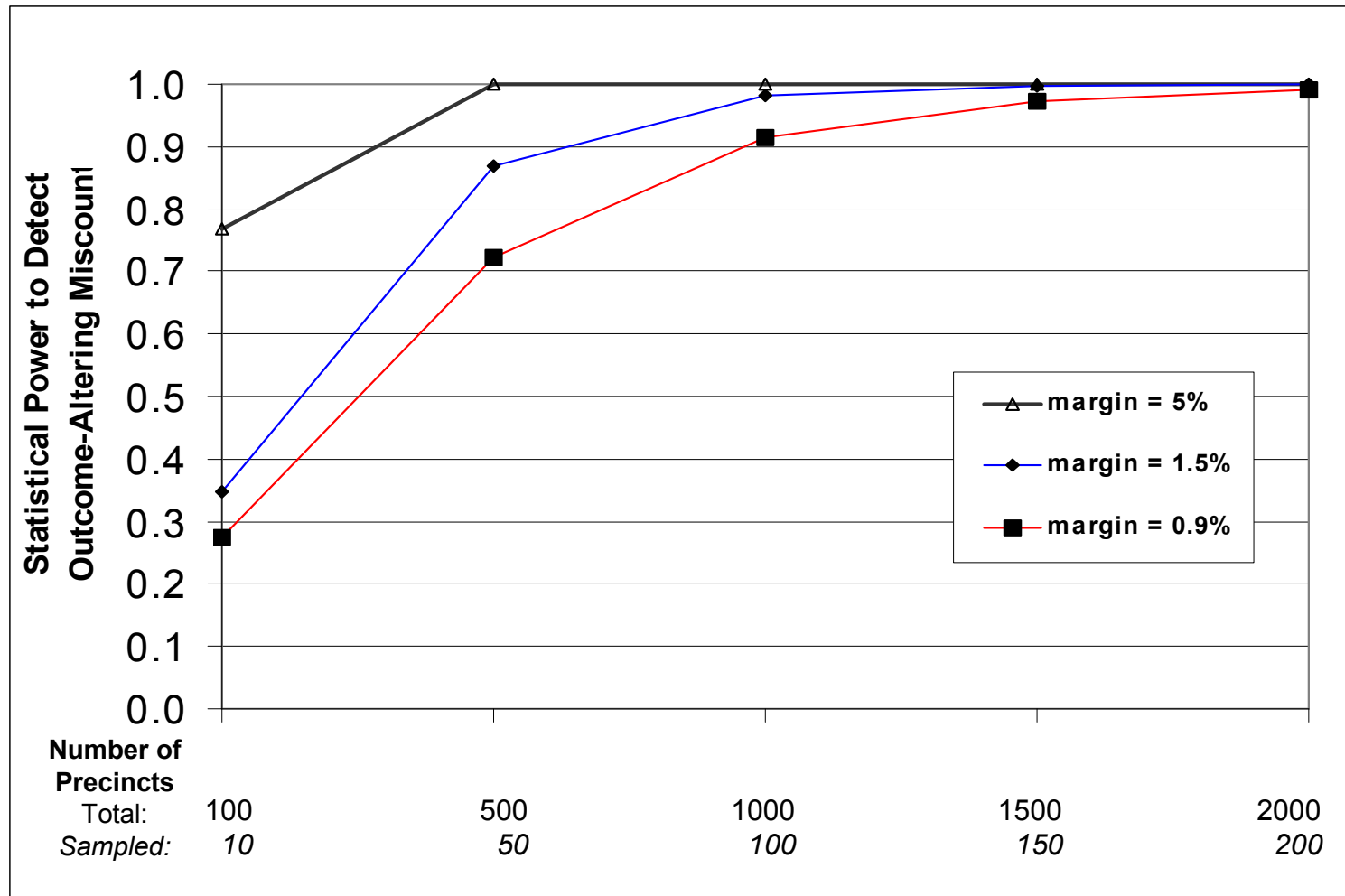
* Results for 1393 election contests for U.S. president, Senate, and House of Representatives. Vote margins were calculated from FEC data for 2002 and 2004; Dr. Adam Carr's Psephos archive for 2006. Numbers of precincts per election were estimated based on the 2004 EAC Election Day Survey¹¹; for House elections, the number of precincts in each state was divided by the number of Congressional Districts to estimate the precincts per District. If an audit size would otherwise be smaller than one precinct per county, one precinct per county is audited. Power is calculated to protect against miscounts residing in the largest precincts, as described in "Implications of Variations in Precinct Size" above.

Statistical Power of a 10% Audit vs. an Audit of 60 Precincts when 8% of Precincts Have Miscounts: By Jurisdiction Size



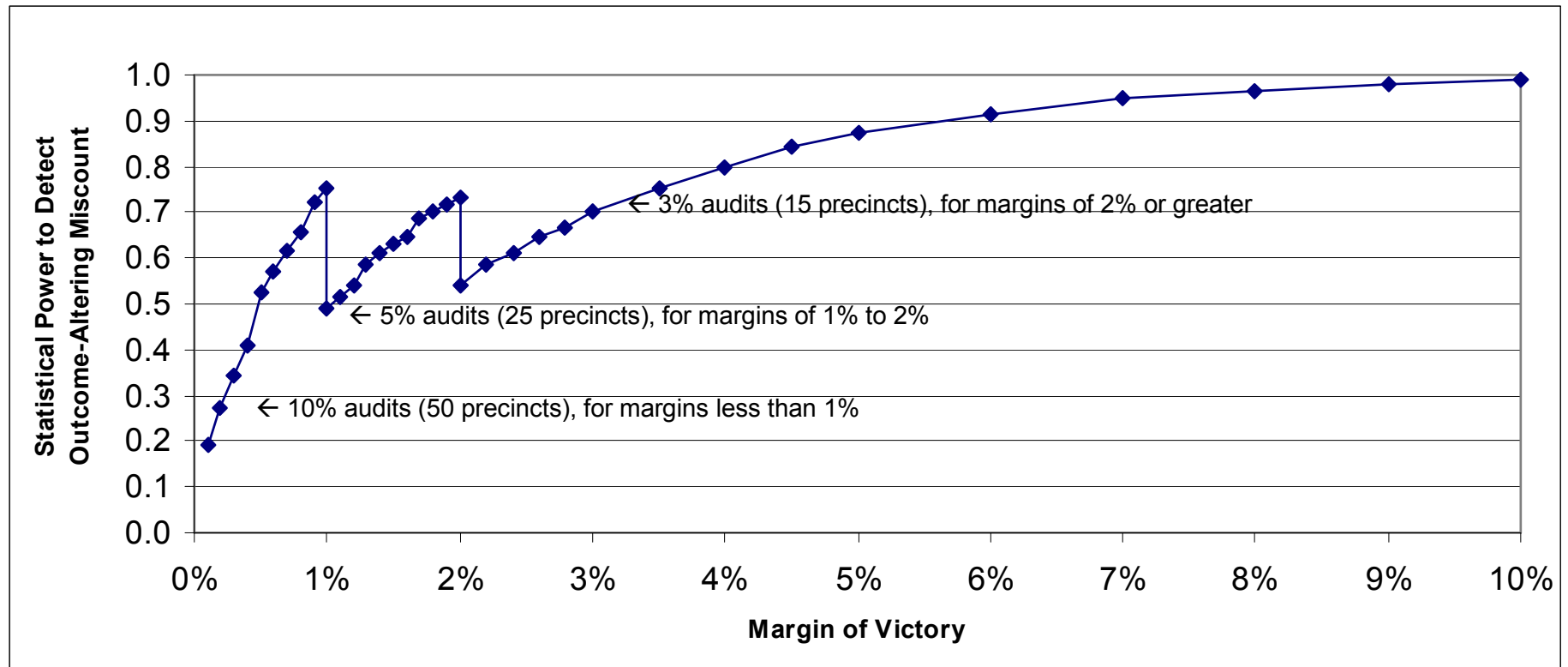
Note that the 60-precinct sample has 99% or greater statistical power *regardless of the total number of precincts*. As the graph suggests, if 8% of the precincts have miscounts, a 60-precinct sample is sufficient

Statistical Power of 10% Audits for Districts: By Number of Precincts Audited and Margin of Victory



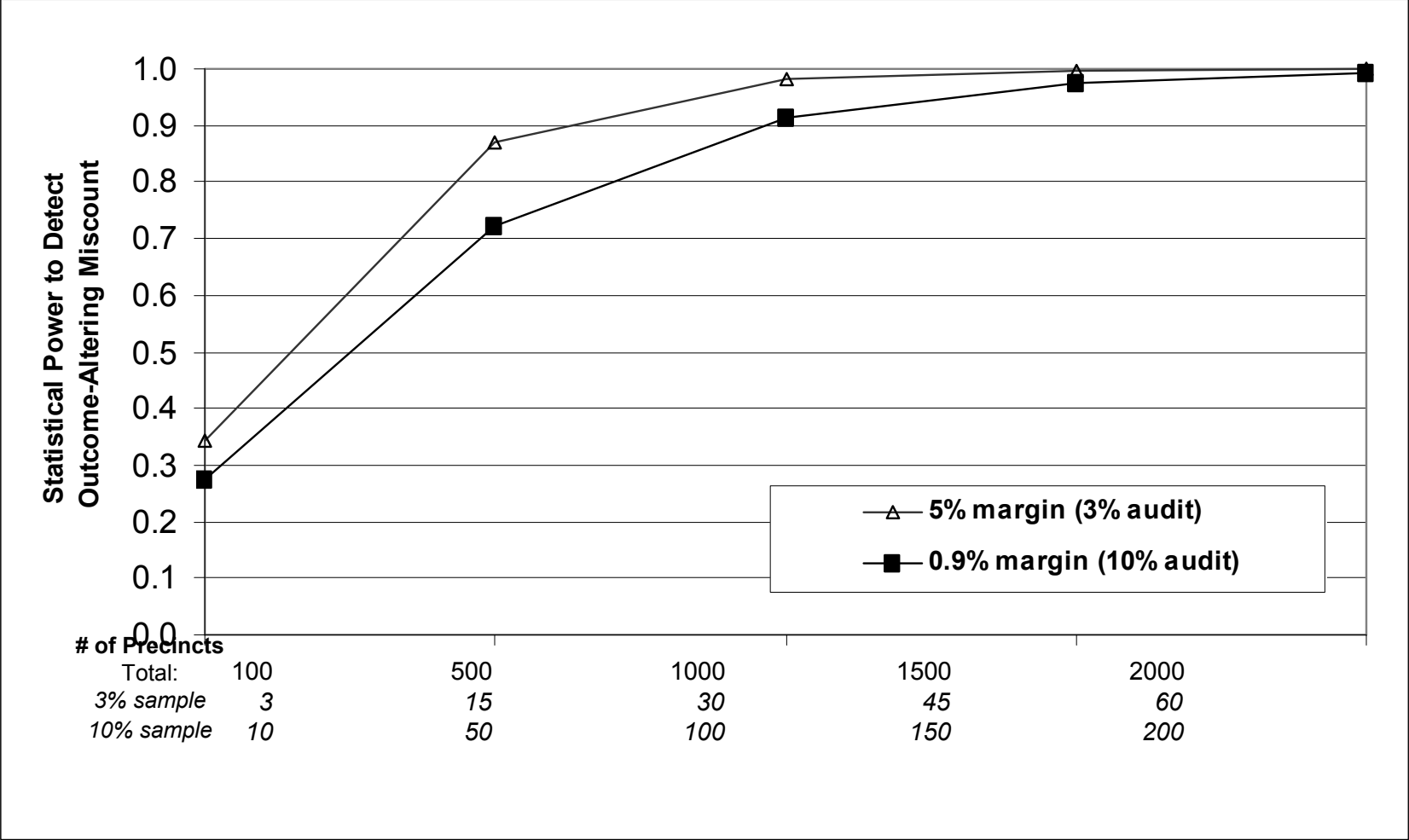
* Power here is calculated as the probability of finding at least one miscounted precinct when the number of miscounted precincts equals the number of average-sized precincts with 20% shifts needed to overturn the election.

Statistical Power of Three-Tiered 3-5-10% Audits in a 500-Precinct Jurisdiction: By Margin of Victory*



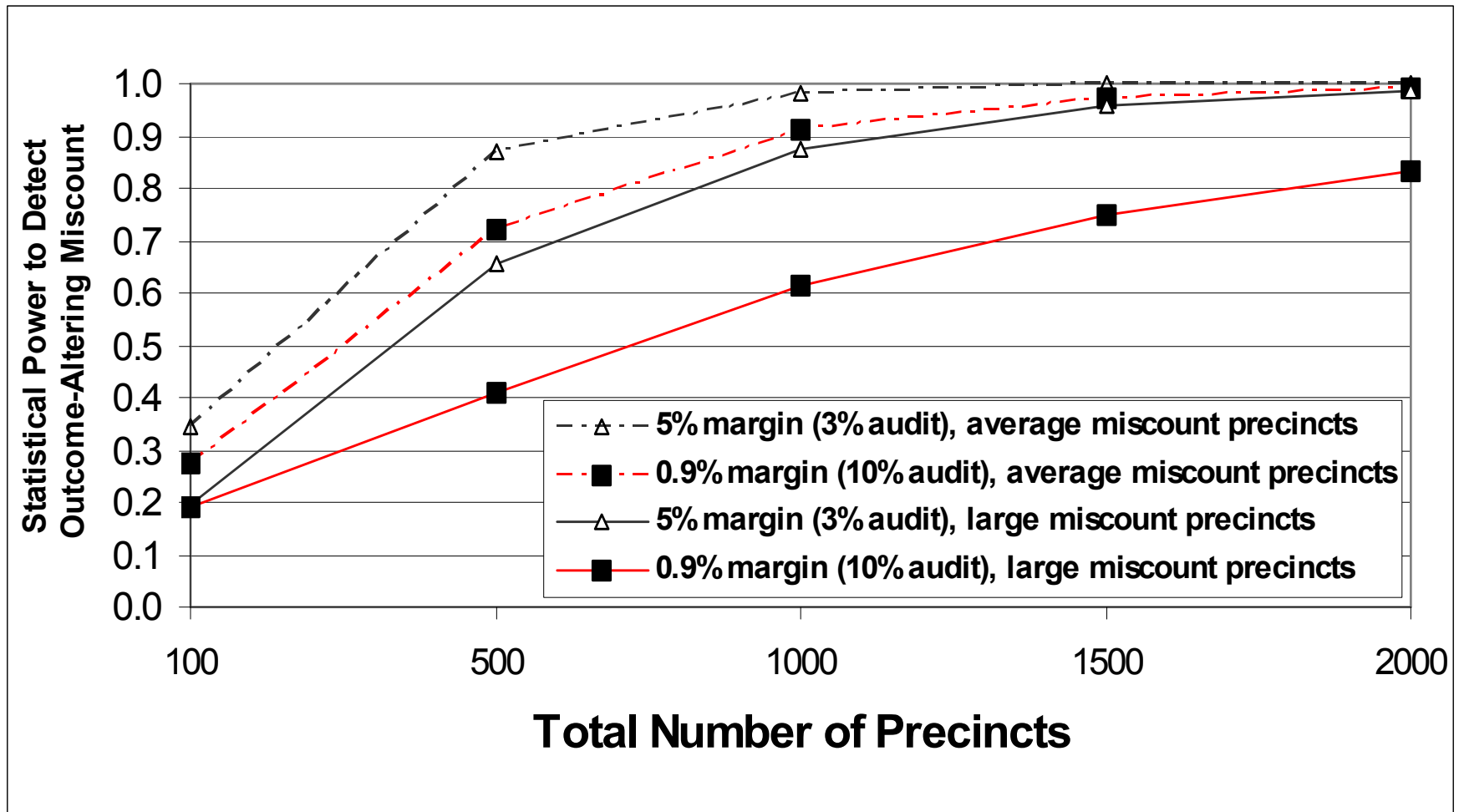
* Assumes the number of precincts with miscounts equals the minimum number needed to overturn the election if all miscounts are in average-sized precincts, each with a vote shift of 20 percentage points.

Statistical Power of Tiered 3% and 10% Auditing: By Jurisdiction Size*



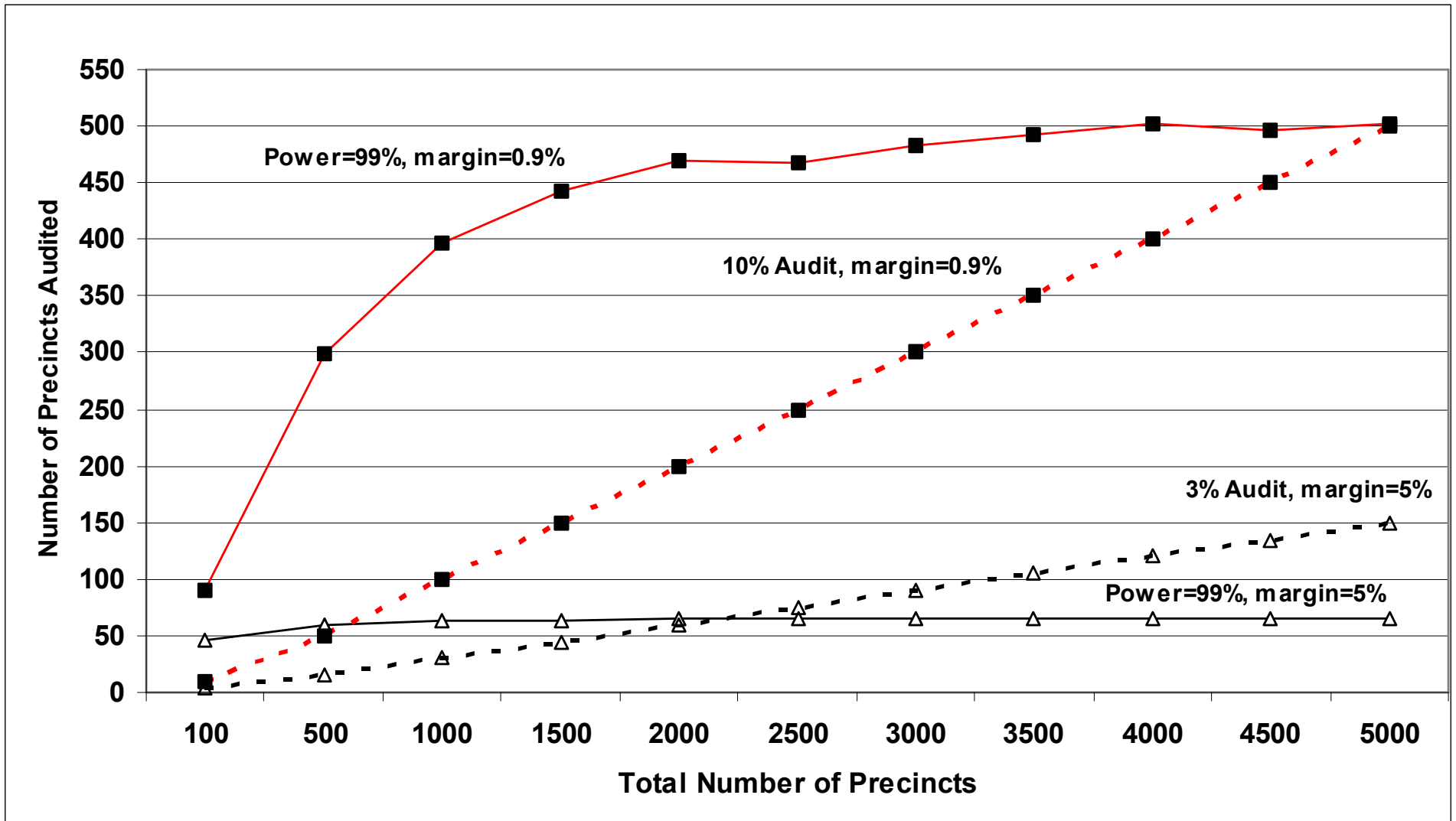
* Assumes miscounts of at most 20% occur in precincts not larger than average.

Power Is Lower for Detecting Miscounts Concentrated in the Largest Precincts



* Power to detect outcome-altering miscounts when they reside in a minimum number of average-sized precincts (“average precincts”) versus when the same number of miscounted votes are in the “largest precincts.” Assumes precinct-size variation as in Figure 5.

Percentage-Based versus SAFE Vote Audit Sample Sizes: By Jurisdiction Size



*Slight “dips” in the SAFE lines are anomalies due to rounding the numbers of sampled precincts up to whole numbers in the formula used for calculating sample sizes